

Title: Plugging into the Best Price

Investigating the effects of slope and y-intercept on the graphs of linear functions.

Link to Outcomes:

- **Problem Solving** Students will examine how varying the slope and y-intercept effects the relationship between cost and time. Given two relationships, students will recognize the more cost efficient.
- **Communications** Students will work cooperatively to develop an understanding of the effects of altering the slope and y-intercept of linear functions.
- **Reasoning** Students will follow logical mathematical directions which will lead them to understanding of the parameters of linear functions.
- **Connections** Students will use knowledge of the slope and y-intercept to solve a real-world problem.
- **Technology** Students will utilize the TI-82 graphics calculator (TI-82) to study the effects of slope and y-intercept on the graph of a linear function.
- **Real-World Applications** Students will demonstrate their ability to apply the concepts of slope and y-intercept to real-world problems.
- **Algebra** Students will use tables and graphs to demonstrate their ability to apply algebraic concepts.

Brief Overview:

Students will use a graphing calculator and the instructional activity sheets to explore the parameters of a linear function and the significance of the slope and y-intercept. This knowledge will be used to interpret a linear relationship between the cost and time of performing a job.

Grade/Level:

Grades 8 – 10, Pre-Algebra and Algebra I

Length/Duration:

Two 90-minute periods or four 50-minute periods

Prerequisite Knowledge:

- Given the graph of a line, determine the slope using rise over run and the y-intercept.
- Plot points on a coordinate plane.
- Graph the line of any equation by generating a table of values.
- Identify the y-intercept from the graph of a line.

Objectives:

- To determine the slope of a line given the graph or the equation using the slope formula.
- To determine the y-intercept of a line given the graph or the equation using the slope formula.
- To interpret a graph of a real-world relationship.

Materials/Resources/Printed Materials:

- Guided Practice Worksheet
- Activity #1
- Activity #2 (Adapted from “The Electrician Problem,” *Mathematics Teacher*, October 1986)
- TI-82 Information Sheet
- TI-82 graphics calculator

Development/Procedures:

1. Students will receive the guided practice sheet, TI-82 graphics calculators, and the TI-82 information sheet.
2. The teacher will lead the class through the guided practice problems including instructions relative to the TI-82 graphics calculator.
3. Students will be arranged in pairs. Each pair will receive activity #1. Students will work cooperatively to complete Activity Sheet #1.
4. Pairs will be joined to form groups of four to discuss findings.
5. Teacher will conduct a whole-class discussion on the effects of slope and y-intercept, emphasizing that slope can be determined by any two points on a line.
6. Students will again work in pairs to complete Activity Sheet #2.
7. The teacher will lead the class in a discussion of group findings.

Evaluation:

The teacher will circulate around the room to be sure teams are on task. Activity Sheets will be collected and graded.

Extension/Follow Up:

1. Determine equations of lines in Activity Sheet #2.
2. Using the last example in Activity Sheet #2, systems of equations may be introduced.
3. Have students measure body parts to determine if there is a relationship between measurements. Use the TI-82 graphics calculator to do best fit lines.
4. Refer to problem #7 on Activity Sheet #2 for a discussion on parallel and perpendicular lines.

5. Definitions can be explored as applicable to real-world problems:

Domain/Range: For example, the domain of a linear function is generally all real numbers. However, when associated with a real-world application, the domain may change.

Independent and Dependent Variables: Cost and Time

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Teacher's Notes

It is recommended that a short lesson introducing the TI-82 and its basic features be presented prior to the Guided Practice Sheet.

Guided Practice Sheet

1. Teacher should lead students in completing the worksheet.
2. The table in #4 and #5 is intended to expose students to the format that will be used in Activity #1.
3. When using the {TRACE} feature on the TI-82, it is important that the {WINDOW} be set to be factors of the number of pixels on the screen (94). This will ensure that students will get x and y values that have only 1 or 2 decimals. The following settings are recommended for these activities:

$$x_{\min} = -11.75$$

$$x_{\max} = 11.75$$

$$x_{\text{scl}} = 1$$

$$y_{\min} = -11.75$$

$$y_{\max} = 11.75$$

$$y_{\text{scl}} = 1$$

Note: After setting the {WINDOW}, do not have students use {ZOOM} {6} because it will reset these values.

4. After completion of the Guided Practice Sheet, guide the class as a whole group in completing the chart for one of the equations.
5. Ask students what they might conclude. For example, no matter which two points students selected, the slope was the same for each line.

Activity #1

1. It is intended that the students give a general sketch of each graph to show the effects the slope has on the appearance of the graph.
2. Question #5 is meant to introduce the concept of the absolute value of the slope which is used to compare steepness. Further discussion may follow.
3. Section C question 2 is intended to have students recognize that the slope is the coefficient of x leading to introduction of the form $y=mx+b$.

Procedures for Graphing Equations and Viewing Tables of Values on the TI-82 Graphics Calculator

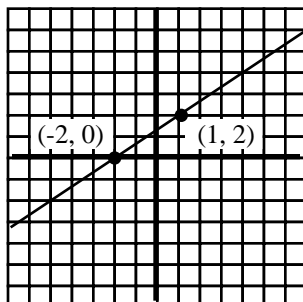
- I. Turn calculator on.
- II. Set up equation in $y =$ editor.
 - a. Press {Y=}
 - b. Clear all equation entries by moving the cursor to each line and press {CLEAR}
 - c. Make sure the cursor is on the $y_1 =$ line
 - d. Enter your equation
 - e. Press {ENTER}
 - f. Enter any other equations in $y_2 =$, $y_3 =$, etc.
- III. Graphing the line
 - a. Press {ZOOM}
 - b. Press {6}
- IV. Use {trace} to find 2 points on the graph
 - a. Press {WINDOW}
 - b. Use the cursor keys to set values to match the following:
xmin = -11.75
xmax = 11.75
xscl = 1
ymin = -11.25
ymax = 11.25
yscl = 1
- V. Viewing the table of values
 - a. Press {2ND} {TBLSET}
 - b. Using the cursor keys set your values to:
TblMin = 0
Tbl = 1
Indpnt: Auto
Depend: Auto
 - c. Press {2ND} {TABLE}
 - d. Use the up/down cursor to view the values in the table.
- VI. Press {2ND} {ON} to turn off the calculator.

Name _____
 Date _____
 Period _____

Guided Practice Worksheet

1. Given the graph below find the slope of the line using $m = \frac{\text{rise}}{\text{run}}$

$m =$ _____



You've seen that the slope of a line can be found graphically by using two points on the line. You can also find the slope algebraically using the same two points on the line by using the following slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

slope formula

For our example, this means

(x_1, y_1)
 $(-2, 0)$

(x_2, y_2)
 $(1, 2)$

OR

(x_1, y_1)
 $(1, 2)$

(x_2, y_2)
 $(-2, 0)$

2. Find the slope using the two points.

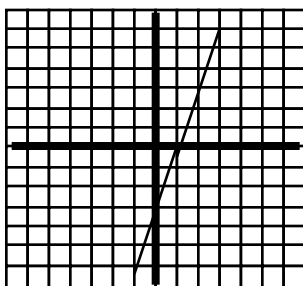
$m =$ _____

$m =$ _____

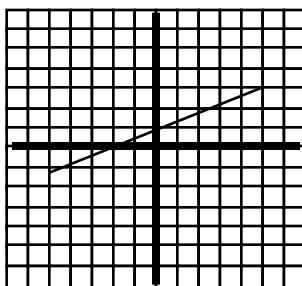
3. What can you conclude from the slopes you found? _____

4. Given the following graphs find the slope of the lines using the slope formula. Fill in the table on the following page using the points found on your graph.

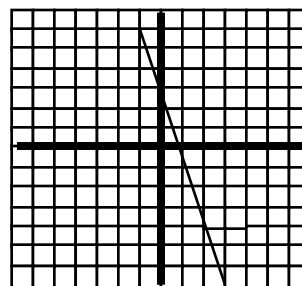
a. $y = 3x - 3$



b. $y = \frac{2}{5}x + 1$



c. $y = -3x + 3$



	x_1	y_1	x_2	y_2	$y_2 - y_1$	$x_2 - x_1$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = 3x - 3$							
$y = \frac{2}{5}x + 1$							
$y = -3x + 3$							

5. Use the TI-82 graphics calculator to verify the slope of the lines in #4. Follow the instruction sheet on the procedures for using the TI-82 table feature to find two ordered pairs on the line. Fill in the table below. Use different ordered pairs than you used in #4.

	x_1	y_1	x_2	y_2	$y_2 - y_1$	$x_2 - x_1$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = 3x - 3$							
$y = \frac{2}{5}x + 1$							
$y = -3x + 3$							

Name _____
 Date _____
 Period _____

ACTIVITY SHEET #1

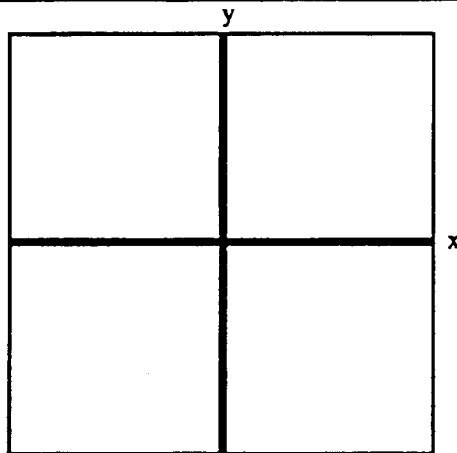
- I. For A, B, C use the TI-82 graphics calculator to graph the given lines. Sketch each line on the given plane. Make sure you label each line. For each line find two points from the table of values found on the calculator. Use these two points to determine the slope of each line, by filling in the table.

A. $y = x$

$y = 2x$

$y = \frac{1}{2}x$

	x_1	y_1	x_2	y_2	$y_2 - y_1$	$x_2 - x_1$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = x$							
$y = 2x$							
$y = \frac{1}{2}x$							



- Which line has the greatest slope value? _____
- Which line has the smallest slope value? _____
- Looking at your sketch, which line is the steepest? _____
- Compare the equation and the calculated slope value. How can you determine the slope of the line by looking at the equation? _____

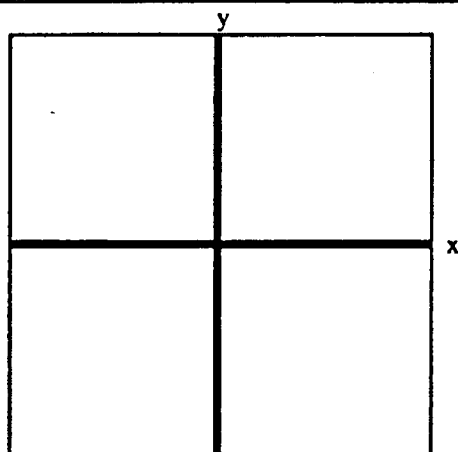
- By looking at the graph of the equations, how is the slope of the line effected by the coefficient of x? _____

B. $y = -x$

$y = -3x$

$y = -\frac{1}{3}x$

	x_1	y_1	x_2	y_2	$y_2 - y_1$	$x_2 - x_1$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = -x$							
$y = -3x$							
$y = -\frac{1}{3}x$							



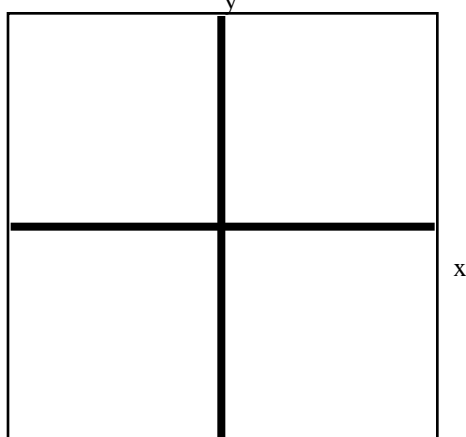
- Which line has the greatest slope value? _____
- Which line has the smallest slope value? _____
- Looking at your sketch, which line is the steepest? _____
- What is the effect of the negative on the graph of the line? _____

- Is the line with the greatest slope also the line with the steepest slope? (Yes/No). _____
Explain. _____

C. $y = x$

$y = x + 3$

$y = x - 4$

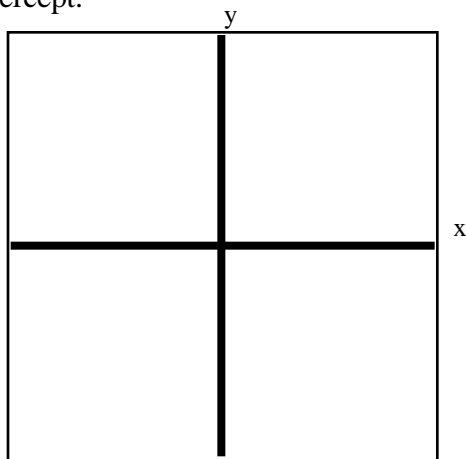


1. What is the y-intercept of

$y = x$	_____
$y = x + 3$	_____
$y = x - 4$	_____
2. What is the slope of

$y = x$	_____
$y = x + 3$	_____
$y = x - 4$	_____
3. Is there a relationship between the y-intercept and the equation of the line? _____

- II. Graph the equation $y = 4x - 1$ on the TI-82 graphics calculator, and sketch below. Label the line and indicate the y-intercept.



1. Determine the slope. $m =$ _____
2. Determine the y-intercept. y-intercept = _____

3. Verify using the TI-82 to find two points from the table of values.

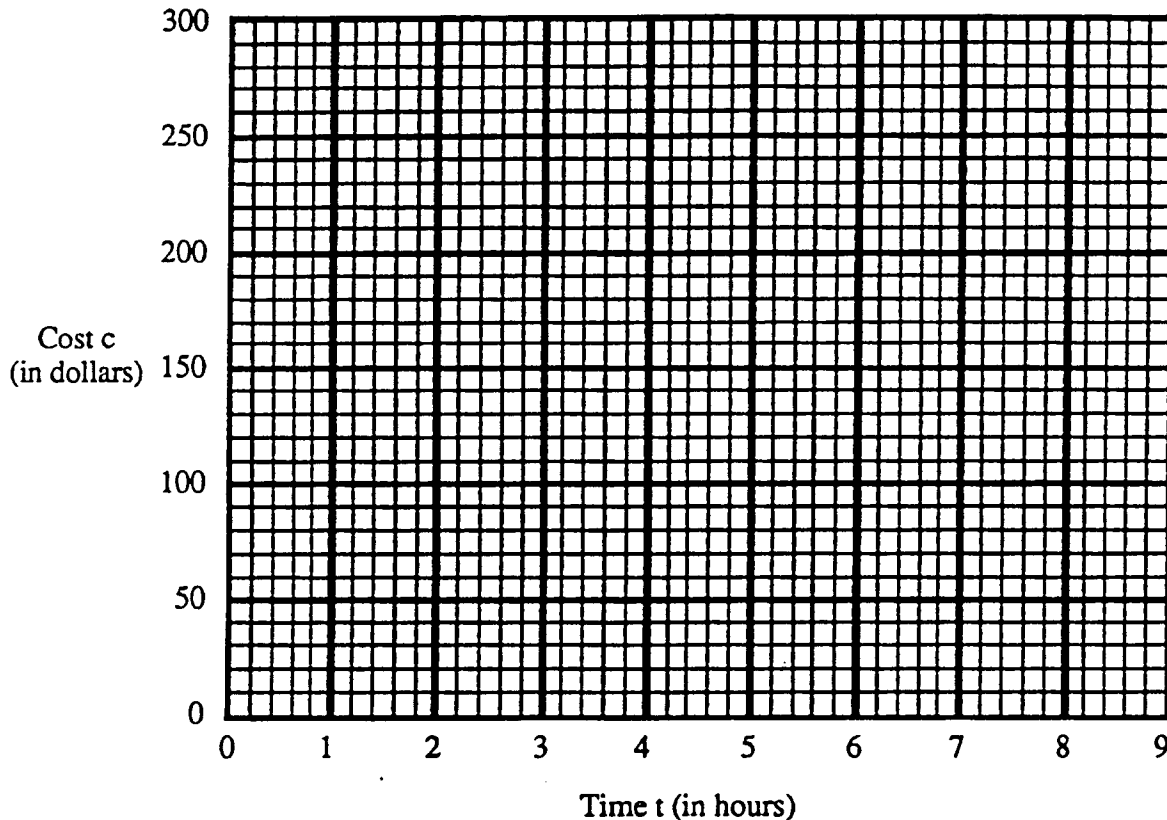
	x_1	y_1	x_2	y_2	$y_2 - y_1$	$x_2 - x_1$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y = 4x - 1$							

Name _____
Date _____
Period _____

ACTIVITY SHEET #2

An electrician's labor fee for a house call is closely related to the amount of time on the job. For example, a job that requires two hours labor will cost the customer \$70. A four-hour job will cost \$120. Let's use this information to answer some questions about the electrician.

Electrician's Fees Graph 1



1. a. Write the data given in the form of ordered pairs (t, c) with job time as the first component, t , and customer cost as the second component, c .
 $(\rule{1cm}{0.4pt}, \rule{1cm}{0.4pt})$ and $(\rule{1cm}{0.4pt}, \rule{1cm}{0.4pt})$
- b. Plot the points represented by these ordered pairs on the coordinate grid provided above.
- c. Use a straightedge to connect the two points on your graph.
- d. What amount does each small division represent on the
x-axis? $\rule{1cm}{0.4pt}$
y-axis? $\rule{1cm}{0.4pt}$

- e. Find the slope (in dollars per hour) between the two points using rise over run. Generate other points on the graph using the slope you determined. Extend the line through all points ensuring the line extends across the entire plane.

Slope = _____

- f. The line you have drawn represents the relationship between the electrician's job time and the customer's job cost. Your line should contain the point (8, 220). Check this! What specific information does the ordered pair (8, 220) supply concerning the electrician's charges? _____
2. Suppose the electrician arrives at a job and the customer explains that he has fixed the problem himself and does not need the electrician's services after all.
- a. Does the graph indicate that the electrician's fee should be \$0? _____
- b. How would you explain this answer? _____

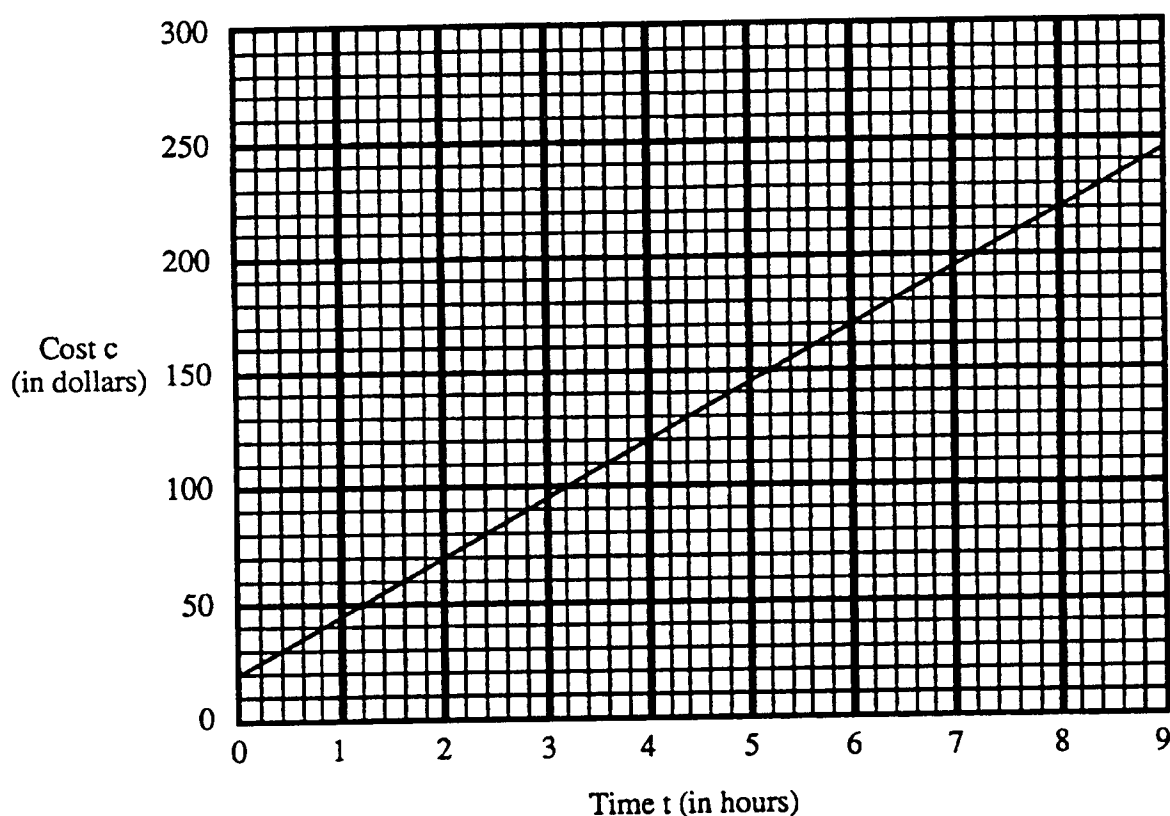
- c. What is the y-intercept of the graph? _____
How does the y-intercept of the graph compare to the 0-hour charge? _____
3. a. What is the difference in cost between a two-hour job and a three-hour job? _____
- b. What is the difference in cost between a one-hour job and a two-hour job? _____
- c. The cost of a six-hour job is \$170. Without referring to your graph, what do you think is the cost of a five-hour job? _____ A seven-hour job? _____
- Compare your answers with the costs shown on your graph.
- d. What is the electrician's per-hour fee (dollars/hour)? _____
How does this compare to the slope you determined in 1e? _____
4. How does the electrician calculate what should be charged for a job? (Hint: Consider your answer to problems 2 and 3d.). _____

5. You have found that the electrician charged a \$20 fee (sometimes called the truck charge) for driving to the work site, plus a \$25 per-hour labor fee. Suppose the electrician raises the per-hour fee to \$30, but uses the same truck charge of \$20. Complete the chart below by determining the cost of each of the following jobs.

	time, t	cost, c	(t, c)
Cost of a two-hour job			
Cost of a five-hour job			
Cost of an eight-hour job			

6. Pictured below is the graph of the original time-cost relationship (\$20 truck charge, \$25 per-hour labor charge).

Electrician's Fees Graph 2

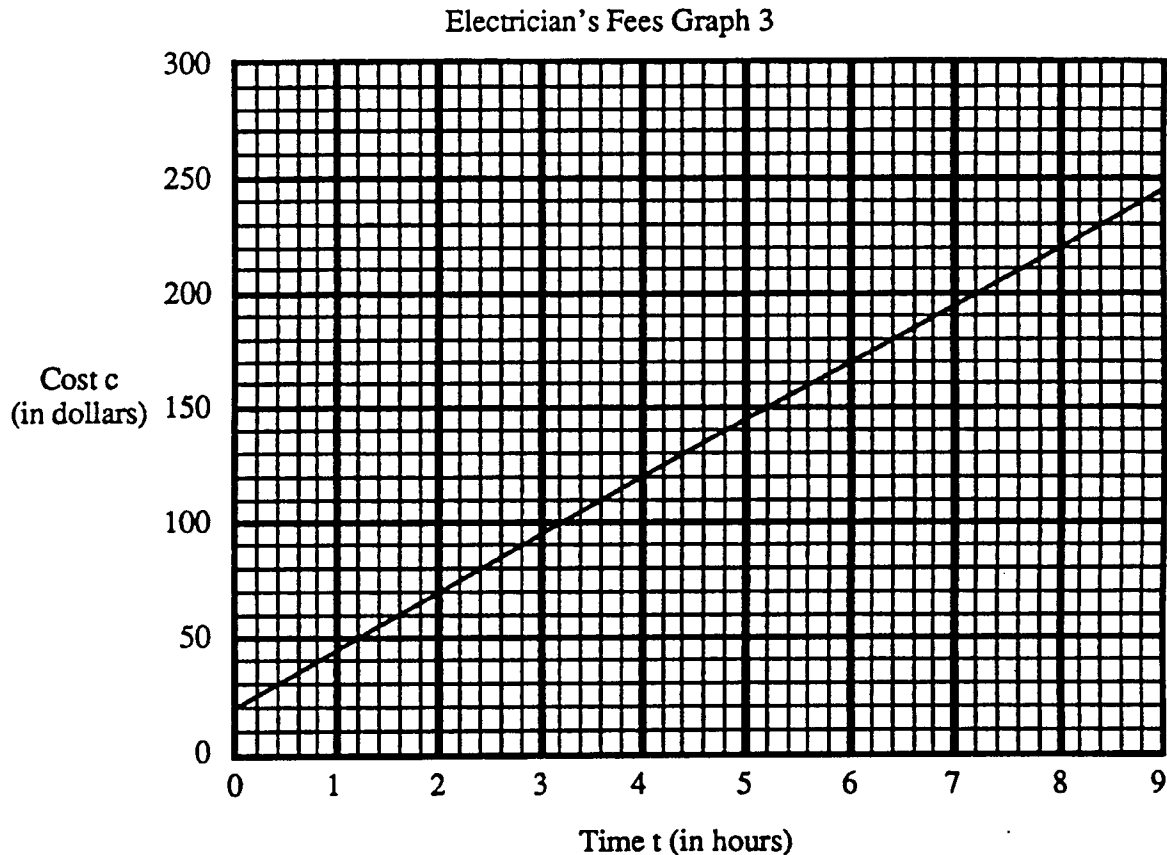


- a. On this same graph, plot the points found in problem 5 and connect them to show the new time-cost relationship (\$20 truck charge, \$30 per-hour labor charge).
- b. Determine the slope of the new line using whatever method you wish. What does this slope represent? _____

- c. When compared to the line representing the original relationship, what feature of this new line shows the increase in the per-hour fee? _____
- d. What feature of the new line shows that the truck charge was not charged? _____
7. Suppose the electrician raises the per-hour fee to \$30, but uses the same truck charge of \$20. Complete the chart below by determining the cost of each of the following jobs.

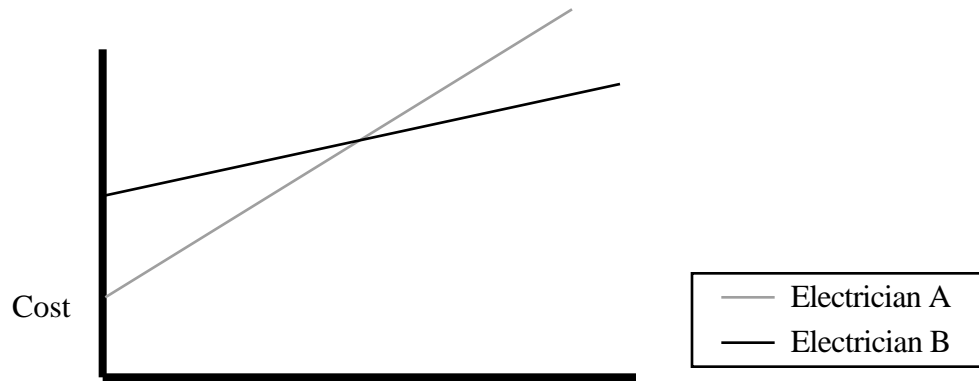
	time, t	cost, c	(t, c)
Cost of a two-hour job			
Cost of a five-hour job			
Cost of an eight-hour job			

8. Pictured below is the graph of the original time-cost relationship (\$20 truck charge, \$25 per-hour labor charge).



- b. What do you think is the slope of the new line? _____
- c. When compared to the line representing the original relationship, what feature of this new line shows the increase in the truck charge? _____

- d. What feature of the new line shows that the per-hour fee was not charged? _____
8. The graph below pictures the time-cost relationships for the fees charged by two different electricians, A and B.



- a. Which electrician has the higher per-hour fee? _____ Explain. _____
- b. Which electrician has the higher truck charge? _____ Explain. _____
- c. Which electrician would you hire (assuming you want your job cost to be as low as possible)? Explain. _____